the relation between σ and ε was chosen to be linear, $\sigma = \sigma_{max}\varepsilon$, which, according to [9], is characteristic for friable media, in particular for sand. Figure 1 shows the dependence of the critical pressure $p_0 = pE^{-1}$ on the yield point k_0 for values of Poisson's ratio of $\nu = 0, 0.5$ and rate of dilatancy $\alpha = 0.1, 0.4$, and 0.7, characteristic for friable media (sand, gravel, etc).

A calculation has shown that the effect of ν and α within the above-stated limits on the magnitude of the critical force is significant. However, the arbitrary values of the critical loadings obtained in this case are unreal and, consequently, no surface instability is observed in practice.

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VARIATIONAL METHOD FOR SOLVING PROBLEMS OF THE PLASTICITY OF COMPRESSIBLE MEDIA

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UDC 539.37

Variational methods used in the theory of plastic flow are formulated on the assumption of the incompressibility of the deformable medium. In solving problems of the mechanics of soils and friable media and technological problems of the plastic shaping of uncompacted materials it is very important to take account of irreversible volumetric change. Extremum and variational theorems are proved in [1, 2] for rigid-plastic and viscoplastic expanding bodies. A variational equation equivalent to a complete system of differential equations is derived for a compressible plastic body.

We consider a material medium with the equations of state

$$S_{ij} = 2g_1(\sigma, H)\varepsilon^*_{ij}, \quad \rho = \varphi(\sigma), \quad \varepsilon^*_{ij} = \varepsilon_{ij} - \frac{1}{3}\varepsilon\delta_{ij}, \tag{1}$$

where the S_{ij} and the ε^*_{ij} are, respectively, the components of the stress deviators and the strain rates; $g_1(\sigma, H)$ and $\varphi(\sigma)$ are functions of the material; ρ is the density of the medium; H is the intensity of shear strain rates; and σ is the mean stress.

Perm'. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 5, pp. 153-155, September-October, 1977. Original article submitted August 10, 1976.

 0021-8944/77/1805-0716$07.50

In accord with the principle of a possible change of the state of strain it was established in [2] that the actual rates of displacement v_i of particles of a compressible medium minimize the functional

$$\Phi = \int \left[\int_0^H g_1(\sigma, H) H dH - \frac{\sigma}{\rho} \rho_{,i} v_i \right] d\omega - \int p_i v_i dS_p \tag{2}$$

with the nonholomonic constraint

$$\rho_{,i} v_i + \rho v_{i,i} = 0, \tag{3}$$

where the p_i are given surface loads on the part of the surface of the body S_p and ω is the fixed volume of the body.

The sufficient condition for a minimum of the functional Φ is proved in [2].

We reduce the problem of the search for a relative minimum of the functional (2) with the equation of constraint (3) to the problem of seeking an absolute extremum. To do this we consider the extended functional

$$\Phi^* = \Phi + \int \theta (\rho v_i)_{,i} d\omega, \tag{4}$$

where θ is a Lagrangian undetermined multiplier.

The principle of the maximum rate of dissipation of energy for a hydrostatic loading was formulated in [1] to prove extremum theorems on the plasticity of compressible bodies: For a fixed value of the volumetric strain e (density) of a body for any given value of the rate of change of volume ε, the inequality

$$(\sigma - \sigma^0)\varepsilon \geqslant 0 \ [\varepsilon = \psi_{,\sigma}\sigma, \ e = \psi(\sigma)]$$

holds, where σ is the actual value of the hydrostatic pressure corresponding to the given value of ε, and σ^0 is any possible value of the mean stress which satisfies the inequality $e - \psi(\sigma^0) > 0$.

Based on the formulation of the maximum principle presented, we calculate the first variation of the functional Φ^* for a fixed density by varying the state of strain and the Lagrangian multiplier

$$\delta\Phi^* = \int \left[2g_1(\sigma, H) \overset{*}{\varepsilon}_{ij} \delta \overset{*}{\varepsilon}_{ij} - \frac{\sigma}{\rho} \rho_{,i} \delta v_i + (\rho v_i)_{,i} \delta\theta + \theta(\rho_{,i} \delta v_i + \rho \delta v_{i,i}) \right] d\omega - \int p_i \delta v_i dS_p = 0. \tag{5}$$

Equation (5) is the condition for a stationary value of the functional Φ^*.

It is known [2] that the Euler–Lagrange equations corresponding to the functional (4) are equations of equilibrium (without body forces) written by taking account of (1) and the continuity condition for the medium, and the multiplier θ has the physical meaning of σ/ρ.

Using the equality $\theta = \sigma/\varphi(\sigma)$, we find from (5), after some transformations,

$$\delta\Phi^* = \int \left[2g_1(\sigma, H) \overset{*}{\varepsilon}_{ij} \delta \overset{*}{\varepsilon}_{ij} + \sigma \delta\varepsilon + (\varphi_{,i} v_i + \varphi\varepsilon) \left(\frac{\varphi - \sigma\varphi_{,\sigma}}{\varphi^2} \right) \delta\sigma \right] d\omega - \int p_i \delta v_i dS_p = 0, \quad \varepsilon = v_{i,i}, \quad \delta\theta = \frac{\varphi - \sigma\varphi_{,\sigma}}{\varphi^2} \delta\sigma.$$

By applying direct variational methods to (6) the distribution of the v_i and σ fields in the volume of a compressible body can be obtained.

The methods of [3, 4] can be used to obtain a numerical solution of variational equation (6).

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